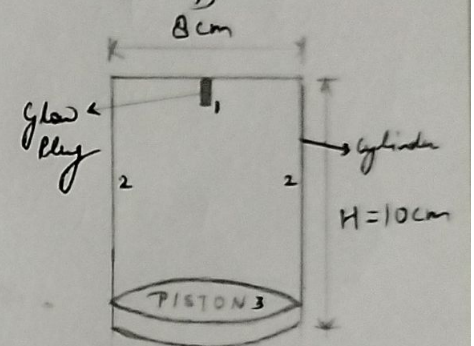
Ques 1.) A glow plug is centrally fitted in a cylinder cover of an engine of 8 cm bore and 10 cm stroke to facilitate starting of an engine. The plug is a piece of nichrome wire 2 mm in diameter and 5 mm in length having $\epsilon = 0.9$ when heated electrically to 1000°C. If the temp. of cylinder and piston is 30°C, emissivity of surface of bore is 0.3 and that of aluminium piston is 0.2. Determine radiation from glow plug which is lost to cylinder and piston crown, assuming vacuum in cylinder during radiation heat transfer.

Sol.) Given :- Temp. of glow Plug, $T_1 = 1000°C$

$T_1 = 1273.15 K$




Temp. of Cylinder wall, $T_2 = 30°C = 303.15 K$

and Temp. of Piston, $T_3 = 303.15 K$

Glow Plug
$d = 2$ mm
$l = 5$ mm

D, Bore = 8cm
H, Stroke = 10cm

$\epsilon_1 = 0.9$
$\epsilon_2 = 0.3$
$\epsilon_3 = 0.2$

At Vacuum, Heat lost by Glow Plug (1) $Q$ = Heat gained by Piston Crown (3) + Heat gained by Cylinder (2)

$= Q_{13} + Q_{12}$

But firstly we need to calculate Shape factors of above bodies.

For Plug & Cylinder

$$F_{12} = \frac{H^2}{R^2+H^2} = \frac{10^2}{4^2+10^2} = 0.862$$

For Plug & Piston crown,

$$F_{13} = \frac{R^2}{R^2+H^2} = \frac{4^2}{4^2+10^2} = 0.1379$$

Calculating Areas,

$A_1$, surface area of Plug $= \frac{\pi d^2}{2} + \pi d l = \frac{\pi (0.002)^2}{2} + \pi (0.002)(0.005)$

$A_3$, surface area of Piston Crown $= \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.08)^2 = 5.024 \times 10^{-3} m^2$

$A_2$, surface area of Cylinder $= \pi D L = \pi (0.08)(0.1) = 0.0251 m^2$

For Plug & Piston, Heat transfer $Q_{13}$, $\left[\sigma = 5.67\times10^{-8}\ W m^{-2} K^{-4}\right]$

$$Q_{13} = \frac{\sigma(T_1^4 - T_3^4)}{\left(\frac{1-\epsilon_1}{A_1\epsilon_1}\right) + \frac{1}{A_1F_{13}} + \left(\frac{1-\epsilon_3}{A_3\epsilon_3}\right)}$$

$$= \frac{5.67\times10^{-8}\left((1273.15)^4 - (303.15)^4\right)}{\left[\frac{1-0.9}{37.7\times10^{-6}\times0.9}\right] + \frac{1}{37.7\times10^{-6}\times0.1379} + \left[\frac{1-0.2}{5.024\times10^{-3}\times0.2}\right]}$$

$$Q_{13} = \frac{148492}{2947.24 + 192351 + 796.178}$$

$$\boxed{Q_{13} = 0.7572\ W}$$

For Plug & cylinder, Heat transfer $Q_{12}$,

$$Q_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1-\epsilon_1}{A_1\epsilon_1}\right) + \frac{1}{A_1F_{12}} + \frac{(1-\epsilon_2)}{A_2\epsilon_2}} \quad [\because T_2 = T_3]$$

$$= \frac{148492}{2947.24 + \frac{10^6}{37.7\times0.862} + \frac{(1-0.3)}{0.3\times0.0251}}$$

$$= \frac{148492}{2947.24 + 30771.69 + 92.961}$$

$$\boxed{Q_{12} = 4.392\ W}$$

Total Heat transferred $= Q_{12} + Q_{13}$

$$Q = 0.7572 + 4.392$$

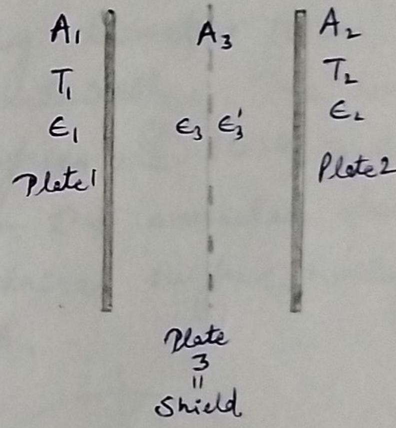
$$\boxed{Q = 5.1492\ W}$$ Ans

Ques 2.) Two Square Plates, each of 5 $m^2$ area are seperated by a gap of 6 mm. One plate, whose surface emissivity is 0·7, is at a temp. of 900 K. The other plate has surface emissivity of 0·95 and a temp. of 300 K. Assuming the plates to be much larger than the gap, calculate net radiation exchange between the plates. If a thin polished metal sheet of surface emissivity 0·15 on both sides is interposed between two plates, calculate steady state temp. and new net radiation exchange through the system. Calculate how many times the heat loss by radiation will be reduced.

Sol 2.) Given :- $T_1 = 900k$ $\epsilon_1 = 0·7$

$T_2 = 300k$ $\epsilon_2 = 0·95$

$\epsilon_3 = \epsilon_3' = 0·15$ $A_1 = A_2 = A_3 = 5m^2$




We know,

$$\left(\frac{q}{A}\right)_{\substack{without\\shield}} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{F_{12}} + \left(\frac{1}{\epsilon_1} - 1\right) + \left(\frac{1}{\epsilon_2} - 1\right)\frac{A_1}{A_2}}$$

Here $F_{12} = 1$, $A_1 = A_2 = A$, we get

$$(Q/A)_{w/o\ shield} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} = \frac{5·67 \times 10^{-8} (900^4 - 300^4)}{\frac{1}{0·7} + \frac{1}{0·95} - 1}$$

$$\left(\frac{q}{A}\right)_{w/o\ shield} = \underline{\underline{24805·3\ W}}$$, Total Heat w/o shield $= 24805·3 \times 4 = \underline{\underline{124\ kW}}$

With shield,

$$\left(\frac{q}{A}\right)_{with\ shield} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1 F_{13}} + \frac{1-\epsilon_3}{A_3\epsilon_3} + \frac{1-\epsilon_3'}{A_3\epsilon_3} + \frac{1}{A_2 F_{23}} + \frac{1-\epsilon_2}{A_2\epsilon_2}}$$

Here, $A_1 = A_2 = A_3 = A$ $F_{13} = F_{23} = 1$ we get,

$$\left(\frac{Q}{A}\right)_{\substack{with \\ shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_2}+\frac{1}{\epsilon_3}+\frac{1}{\epsilon_3'}-2} = \frac{5.67\times10^{-8}(900^4-300^4)}{\frac{1}{0.7}+\frac{1}{0.15}+\frac{1}{0.15}+\frac{1}{0.95}-2}$$

$$\left(\frac{Q}{A}\right)_{\substack{with \\ shield}} = 2659.64\,W$$

Total heat transfered with shield $= 2659.64 \times 5$

$= \underline{\underline{13.298\,kW}}$

$\frac{Q \text{ with shield}}{Q \text{ without shield}} = \frac{13.298}{124} = 0.1072$ hence, heat transfer is reduced to $0.1072$ times.

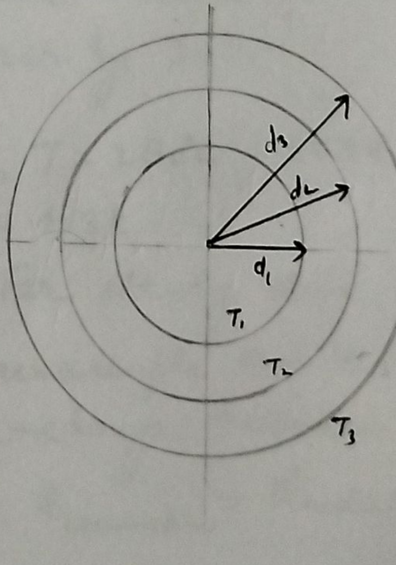
Ques 3.) Three hollow thin cylinders having diameters 10 cm, 20 cm, and 30 cm are arranged concentrically. The temp of innermost and outermost cylindrical surfaces are 100 K and 300 K respectively. Assuming vacuum between the annular spaces, find steady state temp. attained by cylindrical surface having diameter of 20 cm. Take $\epsilon_1 = \epsilon_2 = \epsilon_3 = 0.05$.

Sol.) Given :- $d_1 = 0.1\,m$ $\epsilon_1 = \epsilon_2 = \epsilon_3 = 0.05$

$T_1 = 100\,K$ $d_2 = 0.2\,m$ Steady state

$T_3 = 300\,K$ $d_3 = 0.3\,m$ Heat Transfer

i.e $Q_{12} = Q_{23}$




Finding Areas of cylindrical surfaces,

$A_1 = \pi d_1 L$ (unit length) $A_2 = \pi d_2 L$

$= \pi \times 0.1 \times 1$ $= \pi \times 0.2$

$\underline{A_1 = 0.314\,m^2}$ $\underline{A_2 = 0.628\,m^2}$

$A_3 = \pi d_3 L$ Also, $F_{12} = F_{23} = 1$

$= \pi \times 0.3$

$\underline{A_3 = 0.942\,m^2}$

Equating heat transfer Rates $Q_{12} = Q_{23}$.

$$q_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\epsilon_2}{A_2\epsilon_2}}$$

$$q_{12} = \frac{\sigma(100^4 - T_2^4)}{\frac{1-0.05}{0.314\times 0.05} + \frac{1}{0.314\times 1} + \frac{1-0.05}{0.628\times 0.05}}$$

$$q_{12} = \frac{\sigma(100^4 - T_2^4)}{93.949}$$

$$q_{23} = \frac{\sigma(T_2^4 - T_3^4)}{\frac{1-\epsilon_2}{A_2\epsilon_2} + \frac{1}{A_2F_{23}} + \frac{1-\epsilon_3}{A_3\epsilon_3}}$$

$$q_{23} = \frac{\sigma(T_2^4 - 300^4)}{\frac{1-0.05}{0.628\times 0.05} + \frac{1}{0.628\times 1} + \frac{1-0.05}{0.942\times 0.05}}$$

$$q_{23} = \frac{\sigma(T_2^4 - 300^4)}{52.016}$$

Equating

$$\frac{100^4 - T_2^4}{93.949} = \frac{T_2^4 - 300^4}{52.016}$$, we get $T_2 = 269.17\,k$ Ans

Ques 4.) A thermocouple used to measure temp. of gas flowing through a duct, records 280°C. If the duct wall temp. is 140°C, emissivity of junction is 0.4 and convection coefficient is 150 W/m²K. find true gas temp.. What should be emissivity of junction in order to reduce error by 30%.?

Sol.) Temperature read by thermocouple, $T = 280°C = 553\,K$

Temperature of Duct, $T_w = 140°C = 413\,k$

Let $T_g$ be real gas temperature. For steady state heat transfer, Heat recieved by the thermocouple due to convected gas will be equal to radiant heat exchange between the thermocouple and duct wall. Hence, $q_{convection} = q_{radiation}$

$$h(T_g - T) = \sigma\epsilon(T^4 - T_w^4)$$

$$150(T_g - 553) = 5.67\times 10^{-8} \times 0.4 \times (553^4 - 413^4)$$

$$T_g = 553 + \frac{1461.167}{150}$$

Actual gas Temp. $\boxed{T_g = 562.741\,k}$ Error = 9.45 k.

P.T.O

For 30% error, difference = 6.615K

hence, $T_g = 555.83K$,

Using the same relation, calculating emissivity, $\epsilon'$

$$h(T_g - T) = \sigma \epsilon' (T^4 - T_w^4)$$

$$\frac{150(555.83 - 553)}{5.67\times10^{-8}\times(553^4 - 413^4)} = \epsilon'$$

solving we get

$$\boxed{\epsilon' = 0.116}$$

Ans